

Acceptance Sampling

Acceptance Sampling is a practice whereby a sample is tested from a population (lot), and a decision to **accept or reject** that entire population (lot) is based on the test results of the sample.

acceptance sampling has also become **advantageous** for other companies who have faced destructive inspection, or when the cost associated with 100% inspection was not economical, or where the risk of passing along a defect is low.

Operating Characteristics (OC) Curves

An **OC Curve** is a visual representation of how well a specific sampling plan can discern between good and bad lots.

The **Y-axis is the Probability of Acceptance (P_a)**, and **the curve shows how the probability of acceptance changes depending on the incoming quality level**, which is shown on the X-axis.

An OC Curve allows you to visually evaluate a sampling plan and understand the probability of accepting a lot of varying quality levels.

OC Curves are constructed through a **calculation of the Probability of Acceptance (P_a)** for various **incoming quality levels (p)**.

The majority of acceptance sampling plans use **attribute data**, and this example above is no different.

Because we're using attribute data, the **binomial distribution** can be used to calculate the probability of acceptance. This equation can be executed at various incoming quality levels (p) to construct the entire OC Curve:

$$\text{Binomial Probability Equation: } P(d \leq c) = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

Where:

- d = the number of non-conformances observed during sampling
- c = the acceptance number associated with the sampling plan
- n = the sample size associated with the sampling plan
- p = the incoming quality level

Sampling plans and their associated OC Curves are defined by two parameters, the **Sample Size (n)**, and the **Acceptance Number (c)**.

The **Sample Size (n)** is the number of samples to be inspected. The **Acceptance Number (c)** is the maximum number of non-conformances allowed within the sample.

The last parameter of any sampling plan is the **number of non-conformances, d** , which is the **actual number of non-conformances observed in a sample**.

The 2 Risks of Acceptance Sampling

Producers Risk is the risk (probability) of **rejecting a lot** whose true quality level meets or exceeds the required quality level (AQL). **Producers risk** is typically denoted as α , and the most common target for producers' risk is 5% (0.05).

Consumers Risk is the risk (probability) of **accepting a lot** whose true quality level does not meet the required quality level (LTPD). **Consumers risk** is typically denoted as β , and the most common target for consumers risk is 10% (0.10).

		Acceptance Sampling Results	
		Accept	Reject
Actual Quality of Incoming Product	Good	✓	✗
	Bad	✗	✓

Accepting The Good

Accepting The Bad Consumers Risk, β

Rejecting The Good Producers Risk, α

Rejecting The Bad

For any given sampling plan, the risks associated with that plan (Consumer Risk and Producer Risk) associated with that plan can be seen in the OC Curve along with their relationship to the AQL and LTPD levels.

Key Term within Acceptance Sampling

AQL – Acceptance Quality Limit

AQL is commonly defined as the worst tolerable process average that is still considered acceptable.

AQL is expressed in terms of **percentage of non-conforming material**. So, an AQL of 1.0 means that consumer wants to receive material from a vendor whose process averages, at worst, 1% non-conformances or 1 in 100.

Choosing an AQL for your sampling plan should be risk-based and should take into consideration the **criticality of the quality attribute** being measured, and the **criticality of any associated non-conformances (Critical, Major, Minor)**.

You can designate an **AQL level** for a group of non-conformances (various & multiple failure modes), or you can specify **unique AQL levels for individual non-conformance types** (failure modes).

LTPD – Lot Tolerance Percent Defective

LTPD is another common metric in acceptance sampling and **it reflects the quality limit, in percent defective, that is the poorest quality that can be tolerated.**

LTPD is related to the consumer risk, and typically represents the percent defective that is associated with a 10% probability of acceptance.

The OC Curve also helps to illustrate the consumer and producer risk associated with a particular sampling plan, along with AQL and LTPD

The producers Risk is related to AQL, and is traditionally set at 5% (95% probability of acceptance + 5% probability of rejection),

The consumer risk is tied to LTPD and is set at 10% (10% probability of acceptance, 90% probability of rejection).

AOQ – Average Outgoing Quality

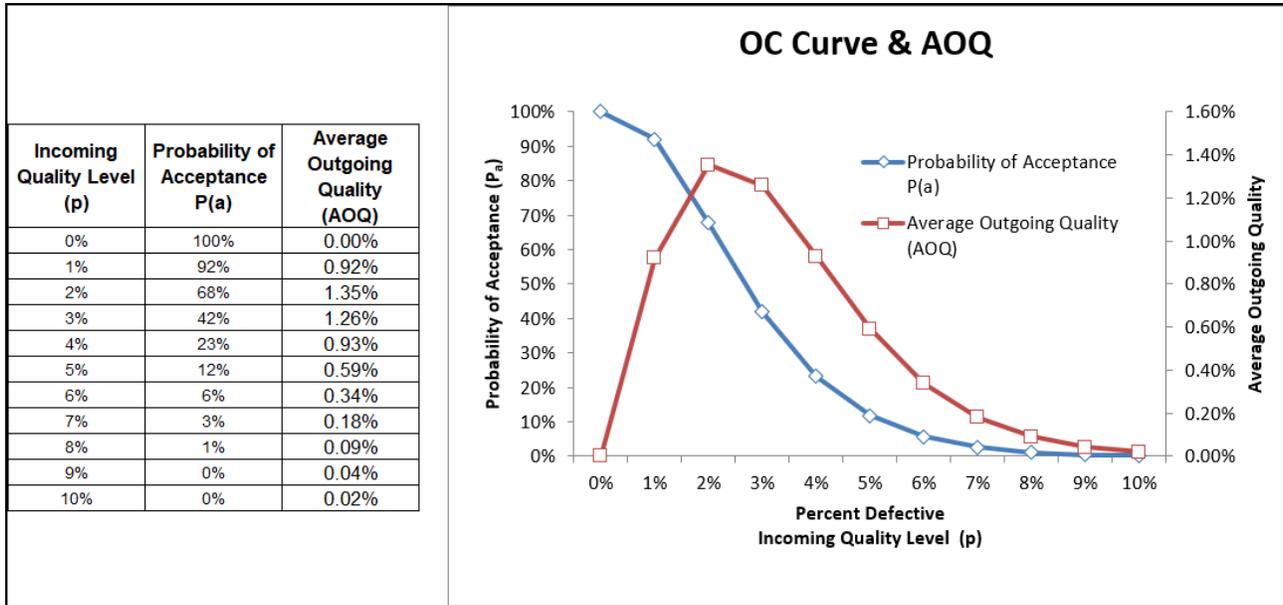
The concept of AOQ is based on the assumption that any lot that gets rejected will be 100% sorted and any non-conformances will be replaced by a conforming unit.

AOQ is defined as a measure of the average outgoing quality of a given sampling plan, in % non-conforming, after lots have been accepted or rejected and 100% sorted.

We can calculate the Average Outgoing Quality using the following Equation:

$$\text{Average Outgoing Quality: } AOQ = P_a * p$$

Essentially, we're multiplying the probability of acceptance (P_a), by the incoming quality level (p), which you can see below.



AOQ tends to peak when the incoming quality level is marginal as we start to accept lots of marginal quality. Then AOQ improves as the incoming quality level deteriorates because the incoming lots get rejected and 100% sorted.

AOQL – Average Outgoing Quality Limit

AOQL is the maximum AOQ for a given sampling plan and reflects the worst possible average outgoing quality associated with a given sampling plan.

Part 2 - Sampling Standards and Plans

Single Sampling Plans

A single sampling plan is the easiest and most common sampling plan where an entire lot is accepted or rejected based on the inspection results of a single sample group of size n , taken from the entire lot (population - N).

Single sampling plans are defined by **three parameters**:

N = Total Lot Size, n = sample size, c = the acceptance number (derived from AQL)

The **total lot size** is designed as N , and the **Sample Size (n)** is the number of samples to be inspected.

The **Acceptance Number (c)** is the maximum number of non-conformances allowed within the sample. **The acceptance number is derived from a combination of the AQL you select for the attribute being inspected for, and the sample size.**

The last **key parameter** of any sampling plan is the **number of non-conformances**, d . This is the actual number of non-conformances observed in a sample.

The single sampling plan is the simplest to administer and execute, however it results in the largest **Average Sampling Number** of all the various plans.

Double Sampling Plans

While a single sampling plan is executed with only a single sample of units, in double sampling, you can take up to two different samples.

Double sampling plans are effective because oftentimes an incoming lot of product **can be so good or so bad** that we can **make a reasonable conclusion about its quality** by taking a sample size much smaller than what is required in a single sampling plan.

Double sampling plans are defined by n_1 , c_1 , r_1 , and n_2 , c_2 , r_2 . The 1 & 2 subscripts (n_1 v n_2) simply denote the 1st and 2nd sample.

n = sample size, c = acceptance number, r = rejection number

A lot can be accepted within the 1st sample if the actual number of non-conformances is less than the acceptance number (c). Similarly, a lot can be rejected if the actual number of non-conformances is greater than the reject number.

If the number of non-conformances observed is greater than that the **1st acceptance number (c_1)** but less than the **1st rejection number (r_1)** then the sampling continues with the second sample, n_2 .

Multiple Sampling Plans

A multiple sampling plan is simply an extension of the double sampling plan, where sampling can go on for up **7 to different samples**. Similar to the double sampling plan, each sample taken has its own accept(c) and reject (r) requirements.

Similarly, to the double sampling plan, over time, these plans have shown to have a lower **average sampling number (ASN)**, when compared to a double or single sampling plan.

ANSI/ASQ Z1.4 - The Attribute Sampling Plan

The ANSI/ASQ **Z1.4 standard** is acceptance sampling for **attribute data**.

The Z1.4 standard allows the user to choose between **single, double, or multiple sampling plans**, and then assists the user in setting up their **sampling scheme**.

A **sampling scheme** is a set of sampling plans that are used along with **switching rules** that govern with sampling plan should be used. These various sampling plans include **normal sampling, heightened sampling** and **reduced sampling**.

These switching rules are integral to the standard.

These sampling schemes are meant to reward vendors who have demonstrated a stable, **high quality process** with **reduced sampling**, and **encourage continuous improvement** for poorly performing vendors with **heightened sampling**.

The ANSI standard has **3 general inspection levels (I, II and III)**, and **4 special inspections**.

The **special inspection levels** should only be used when small sample sizes are necessary due to high inspection costs. These small sample sizes naturally **result in large sampling risks** for both the producer and consumer.

Sampling Scheme

All sampling schemes should start with **Normal Inspection**.

Reduced inspection is a set of plans that have a **reduced sample size** compared to normal inspection. Reducing the sample size increases the consumer risk, however this risk can be tolerated due to the strong performance of the supplier.

Tightened inspection is a set of plans that have the **same sample size**, but a **lower acceptance number and rejection number** to **reduce the consumer risk**.

There is also a **fourth condition within the switching scheme** that is worth mentioning which is **discontinued sampling**.

If an incoming lot has been on tightened inspection and has 5 consecutive rejection, the entire sampling methodology can be discontinued which moves the incoming material to 100% inspection. Sampling cannot resume until the vendor has taken corrective actions to improve the stability or capability of their process.

Switching Rules

Normal to Tightened

Switching from **normal sampling** to **tightened sampling** should occur when 2 out of 5 consecutive lots are rejected.

Tightened to Normal

A vendor can come off of tightened inspection and move back to normal inspection after **5 consecutive lots being accepted**.

Normal to Reduced

To go from **normal sampling** to **reduced sampling** all of the follow conditions must be met:

- 10 consecutive lots accepted
- The total number of non-conformances in the previous 10 lots is less than the “Limit Numbers” than the Limit Number found in Table VIII in the applicable standard. If using double or multiple sampling, all inspected should be considered.
- Production is at a continuous and steady rate
- Reduced sampling is acceptable, desirable and approved by the responsible authority

Reduced to Normal

A sampling scheme should switch from reduced inspection to normal inspection if any of the following conditions occur:

- A single lot is rejected
- If the number of non-conformances (d) is between the acceptance number (c) and the rejection number (r). In this scenario, the lot is accepted, but reduced sampling is discontinued.
- The supplier’s production process is irregular or infrequent
- Other conditions warrant a switch

ANSI/ASQ Z1.9 - The Variable Sampling Plan

The **ANSI/ASQ Z1.9 standard** is acceptance sampling for **variable data** (to estimate the lot percent non-conformances).

The advantage of using variable data is that the sample sizes are much smaller when compared to attribute sampling, while still offering the same level of protection against consumer and producer risks. The disadvantage is in the administration of the plans where the math is complex.

These sampling plans are based on the **assumption of normality**, and they use [descriptive statistics](#) like the **average, range** and **standard deviation** to make inferences about the overall population.

Similar to Z1.4, Z1.9 is based on a **switching scheme** where product can switch between **normal, reduced or tightened** inspection based on the incoming quality from the supplier. The switching criteria within Z1.9 closely reflects those in Z.14.

Additionally, Z1.9 also has plans for **single, double and multiple sampling plans**.

Variable sampling often requires comparison of sample statistics (average and standard deviation) against both an **upper and lower specification** to estimate the overall percentage of non-conforming material in lot. This also increases the complexity and administrative costs associated with the variable sampling plan.

This standard provides **3 different calculation processes for sampling** based on 3 different **estimates** of the **Lot Variation**.

1. When **the standard deviation is known**.
2. When the standard deviation is unknown and the **average range** is used to estimate the Variation.
3. When the standard deviation is unknown and the **sample standard deviation** is used to estimate the Variation.

This standard also provides **5 different inspection levels** which can be used. 3 levels of **General Inspection** (I, II and III) and 2 levels of **Special Inspection** (S3 & S4). Similar to the Z1.4 standard, the default inspection level is general level II.

Dodge-Romig Sampling Tables

While the Z1.4 and Z1.9 standards provide sampling plans based on AQL, the **Dodge-Romig sampling tables** provide sampling plans based on a desired **Lot Tolerance Percent Defective (LTPD)**, or **Average Outgoing Quality Limit (AOQL)**.

These plans are specific to **attribute data only**, and come in two varieties - single or double sampling plan.

These plans are created so your LTPD corresponds to a specific **incoming quality level**, say **1% non-conforming** for example. This means that the plan would have a 10% probability of accepting a lot whose quality level is 1% non-conforming.

The **AOQL plans** are designed to ensure that the AOQL will be less than the number specified in the plan.

To properly use a dodge-romig table, you need to know two parameters. The **total lot size** of the incoming material, and the **expected process average of non-conforming material** associated with process being sampled from.

One of the primary benefits of the Dodge-Romig tables is that they tend to minimize the average number of units inspected.

Part 3 - Sample Integrity

Care must be taken during sampling to ensure the samples are representative of the overall population, and not subject to any biases or unnecessary risks.

Randomness in the sampling process is critical. This ensures that the sample is representative of both the central tendency and variation associated with the overall population.

The goal of **random sampling is to ensure that every item in the population has an equal chance of being selected for the sample.**

Sampling without replacement means that when taking multiple groups of samples, you **must not return the tested product back** into the general population of product. This is an important requirement within double or multiple sampling.

Lot or Batch Size			Special Inspection Levels				General Inspection Levels		
			S-1	S-2	S-3	S-4	I	II	III
2	to	8	A	A	A	A	A	B	
9	to	15	A	A	A	A	A	C	
16	to	25	A	A	B	B	B	D	
26	to	50	A	B	B	C	C	E	
51	to	90	B	B	C	C	C	F	
91	to	150	B	B	C	D	D	G	
151	to	280	B	C	D	E	E	H	
281	to	500	B	C	D	E	F	J	
501	to	1200	C	C	E	F	G	K	
1201	to	3200	C	D	E	G	H	L	
3201	to	10000	C	D	F	G	J	M	
10001	to	35000	C	D	F	H	K	N	
35001	to	150000	D	E	G	J	L	P	
150001	to	500000	D	E	G	J	M	Q	
	>	500001	D	E	H	K	N	R	

Attribute Sampling Size Code Letter Table

Sample Size Code Letter	Sample Size	AQL (Acceptance Quality Limit) for Normal Inspection															
		0.25	0.4	0.65	1	1.5	2.5	4	6.5	10	15	25	40	65			
A	2	V		V		V		V		V		V		V		V	
B	3	V		V		V		V		V		V		V		V	
C	5	V		V		V		V		V		V		V		V	
D	8	V		V		V		V		V		V		V		V	
E	13	V		V		V		V		V		V		V		V	
F	20	V		V		V		V		V		V		V		V	
G	32	V		V		V		V		V		V		V		V	
H	50	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
J	80	V		V		V		V		V		V		V		V	
K	125	V		V		V		V		V		V		V		V	
L	200	1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22
M	315	2	3	3	4	5	6	7	8	10	11	14	15	21	22	V	
N	500	3	4	5	6	7	8	10	11	14	15	21	22	V		V	
P	800	5	6	7	8	10	11	14	15	21	22	V		V		V	
Q	1250	7	8	10	11	14	15	21	22	V		V		V		V	
R	2000	10	11	14	15	21	22	V		V		V		V		V	

Single Normal Attribute Sampling Table

